Division of labor in child care: A game-theoretic approach

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Article

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Abstract

This paper uses a game of repeated play to model parental child care in order to examine the gap between the expectations of egalitarian-minded couples before the transition to parenthood and the reality of parenthood, with its gendered roles. This is done first in a gender-free context in order to examine the mechanism by which the division of labor is established in a family – it is this same process through which gendered expectations have an impact. The analysis shows that it is difficult to achieve the equilibrium of equal sharing of child care, even when this is the preference of the parents. This leads to a discussion of alterations and meta-strategies for couples who want to share care equally. Gender differences between parents are also modeled, including the impact these have on outcomes and equilibria.

Keywords

Child care, labor division in families, family work, game theory, gender, parental time, time use

Introduction

In two-parent households with young children in which both parents work full-time, it is well established that women carry out more child care and

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household labor than men (Coltrane, 2000; Hook, 2010; Wilkie et al., 1998). Transition to parenthood increases the work that needs to be done in the home and solidifies the gender imbalance in the division of labor (Sanchez and Thomson, 1997), but it is difficult to quantify the inequalities in parenting and child care, and widely varying estimates circulate about the amount of time mothers and fathers spend on family work (Yeung et al., 2001).

This paper uses a game-theoretic model to analyze the division of child care labor between parents. Economists have modeled economic decisions of families since the 1950's, but until the 1980's families were usually treated as a single unit (Becker, 1981; Samuelson, 1956) with a single utility function common to all family members. Later work (see Lundberg and Pollak, 1996) saw the development of game-theoretic models that assume partners in a marriage seek to maximize their individual utility, and some of the game-theoretic work has addressed the division of household labor (Youm and Laumann, 2003), but none has specifically addressed child care.

Research on division of household labor

After women began to allocate an increasing amount of their time to paid work, social scientists started to look at the impact this workforce change was having on housework and child care. Over the past half-century the hours spent by married couples on housework have decreased, and men have increased their hours spent on household duties; however, women still do more of the housework than men (Bianchi et al., 2000; Coltrane, 2000; Sanchez and Thomson, 1997; Wilkie et al., 1998) In particular, mothers' increasing participation in the workforce has not led to a more equitable division of child care labor (Bianchi, 2000). Research on the division of household labor has sought to explain the persistent gender inequality in child care and routine household chores. Many explanations have been explored in the literature, including gender construction, performance, and ideology (Barnett and Baruch, 1987; Coltrane, 1989; Ross, 1987); economic theory, including relative resources, exchange theory, specialization, and altruism (Becker, 1981; Brines, 1993); time availability and workplace norms and structure (LaRossa, 1983; Nock and Kingston, 1988; Rapoport and Le Bourdais, 2008); power and resource availability (Ross, 1987); and issues pertaining to stages of life and age (Barnett and Baruch, 1987; Coltrane and Ishii-Kuntz, 1992; Suitor, 1991; Torr and Short, 2004).

In fact, the transition to parenthood correlates with a more traditional division of labor, with mothers working fewer hours outside the home and fathers working greater hours outside the home. In some families, this transition and the accompanying traditional division of housekeeping and child care labor is a conscious choice, with fathers and mothers having different expectations around paid work and the work of childrearing. This can lead to more traditional marriages or to satisfaction and perceived fairness by mothers who work outside the home but still contribute disproportionately to child care (Wilkie et al., 1998). In other families, however, both parents have a stated preference for sharing in the care of children (and in paid work) – but when faced with the reality of parenthood, the couple finds they are not sharing equally (Deutsch, 1999; Wilkie et al., 1998) and that the mother is more responsible for child care, regardless of the amount of paid work she does (Hook, 2010).

It is clear that while ideology may have an influence on parents' behavior, it does not determine the division of labor (Ishii-Kuntz and Coltrane, 1992; Sanchez and Thomson, 1997). This leads to the question of why ideology and attitudes about housework and child care do not have a greater impact on the division of labor. The situation is further complicated by the fact that while women reduce time spent on housework as their paid employment goes up, they do not appear to reduce the amount of time spent on child care (Bianchi, 2000; Nock and Kingston, 1988). Gendered expectations certainly have an impact here. Traditional images of motherhood include the idea that mothers derive their primary meaning in life from that role, sacrifice things for their children, and hold their children's fate in their hands (Thompson and Walker, 1989). Traditional images of fathers include the idea that they support their families through paid work and have neither an interest in, nor aptitude for, day-to-day child care (Thompson and Walker, 1989). However, these images and the gendered expectations that go along with them may be changing (Deutsch, 1999; Plant, 2010; Smith, 2009).

Modeling division of child care

This paper examines the gap between the expectations of egalitarian-minded couples before the transition to parenthood and the reality of parenthood, with its gendered roles. The focus of this paper is modeling couples who intend, before the onset of parenthood, to share child care duties rather than making use of specialization and exchanges between market work and child care, as in a more traditional marriage. Child care in such a two-parent family will be modeled as a repeated game between two players. This analysis is outside the framework of gender and relies only on identifying the tasks to be done and the goals of the parents. This simple model reveals one method by which parents might settle into a division of labor, and it is this same process through which gendered expectations have an impact. The initial focus of this paper will be on modeling a situation in which parental goals are identical and include equal sharing of child care, in order to understand why even parents with a thoroughly egalitarian relationship might end up with a division of labor at odds with their goal of shared care. Along the way, aspects of the game that pertain to gender roles and the current state of gendered division of labor in childrearing will be analyzed. I will then explore variations of this game in which parental goals are altered, first to attempt to make it easier to achieve an equal distribution of care and then to explore how the game changes when parental goals are no longer identical.

Child care game

Consider a situation in which there are a certain number of hours of child care that need to be provided by a mother and a father, modeled as a two-player game between the parents. The following goals are assigned to each parent:

G1. **Sufficient care.** Provide enough care for the child.
G2. **Quality time.** Have enough time with the child to establish and maintain a bond. This could be either time alone with the child or time with the other parent present.
G3. **Personal time.** Put in as little time as possible on child care in order to have time free for other activities, including recreation and paid work.
G4. **Equal sharing.** Provide the same number of hours of care as the other parent.

Each parent can choose from a set of different strategies, corresponding to the number of hours of child care each will provide. To keep the choices relatively simple, each parent will provide between zero hours and the number of hours needed (established by goal G1) in increments of a half-hour. The parents' goals are used to establish cardinal utility functions, which give not just a ranking of the outcomes, but also the degree of their preference. The following parameters are used in the utility functions:

$g_{P,1}$	Importance of G1 (sufficient care) on $0-10$ scale for parent P
$h_{P,1}$	Total combined hours required to meet G1 for parent P
$g_{P,2}$	Importance of G2 (quality time) on $0-10$ scale for parent P
$h_{P,2}$	Hours required to meet G2 for parent P
$g_{P,3}$	Importance of G3 (personal time) on $0-10$ scale for parent P
$g_{P,4}$	Importance of G4 (shared care) on $0-10$ scale for parent P

Here $P=r$ indicates the parent whose strategies are represented by rows (in this game that is the father) and $P=c$ indicates the column player (here the mother). Note that the importance of the four goals can be reordered for a parent simply by changing the parameters $g_{P,i}$. Also note that one restriction of this model is the assumption is that the hours in $h_{P,1}$ must be met by the parents, rather than by a non-parental child care provider.

Derivation of payoff functions

The parameters determine cardinal utility functions for each parent for each possible pair of strategies by the two parents. If $u_{P,g}(i,j)$ is the payoff to parent P for meeting goal g when parent r does i hours of care and parent c does j hours of care, then the utility function for parent P is given by summing up the utility gained by meeting (or failing to meet) each of the four goals.

$$u_P(i,j)=\sum_{g=1}^{4} u_{P,g}(i,j) \tag{1}$$

The four utility sub-functions are constructed in a straight-forward manner. For example, consider the utility sub-function based on goal G1 (sufficient care). The maximum utility of $g_{P,1}$ is given any time the total hour criteria is met, so $u_{P,1}(i,j)=g_{P,1}$ if $i+j \geq h_{P,1}$. If $i+j<h_{P,1}$, then parent P is meeting

$$\frac{i+j}{h_{P,1}}\text{percent} \tag{2}$$

of the goal, so multiplying this percentage by $g_{P,1}$ will give the utility for parent P. The other three utility sub-functions are constructed similarly. Note that the utility for G2 (quality time) counts only the hours of one parent and maximum utility is achieved when the goal of $h_{P,2}$ hours is met. Goal G3 (personal time) is best met by doing zero hours of care (although this could negatively impact other goals) and the utility function is linearly scaled so that a parent doing $h_{P,1}$ hours gets no utility from this goal. Goal

G4 (shared care) is best met when $i=j$, and is linearly scaled until the payoff is zero when $|i-j|=h_{P,1}$. The payoff functions are as follows:

$$u_{P,1}(i,j)=g_{P,1}\left(\frac{\min(i+j,h_{P,1})}{h_{P,1}}\right) \tag{3}$$

$$u_{r,2}(i,j)=g_{r,2}\left(\frac{\min(i,h_{r,2})}{h_{r,2}}\right) \tag{4}$$

$$u_{c,2}(i,j)=g_{c,2}\left(\frac{\min(j,h_{c,2})}{h_{c,2}}\right) \tag{5}$$

$$u_{r,3}(i,j)=g_{r,3}\max\left(1-\frac{i}{h_{r,1}},0\right) \tag{6}$$

$$u_{c,3}(i,j)=g_{c,3}\max\left(1-\frac{j}{h_{c,1}},0\right) \tag{7}$$

$$u_{P,4}(i,j)=g_{P,4}\max\left(1-\frac{|i-j|}{h_{P,1}},0\right) \tag{8}$$

I do not assume that the goals, outcomes, or structure of the game are common knowledge to both parties. In fact, a parent may not be entirely clear about his or her own goals. I do assume, however, that given a fixed number of hours of child care provided by a partner, the parent will play a strategy that gets him or her the highest payoff possible. In other words, the mother will play a strategy that gets her the highest payoff in a fixed row, and the father will play a strategy that gets him the highest payoff in a fixed column. I assume that this is strictly a non-cooperative game and no enforceable negotiation is allowed between the parents (Youm and Laumann, 2003).

Symmetric child care game

In the symmetric game in which the players' goals are identical, the following values are assigned to the parameters.

$$\begin{array}{ll} g_{P,1}=10 & h_{P,1}=3 \\ g_{P,2}=3 & h_{P,2}=1 \\ g_{P,3}=2 & \\ g_{P,4}=1 & \end{array} \tag{9}$$

These parameters order the importance of the goals in the order originally given and indicate that there are three hours of care to be provided and that each parent, to be happy with the amount of time they have to bond with their child, wants to provide at least one hour of care. Note that goal G1 (sufficient care) is much more important than the remaining goals in this parameter scheme, and the results of this game are robust if goal G1 is important enough. Specifically, if $g_{P,1} > g_{P,3} + g_{P,4}$ and $h_{P,1}$ and $h_{P,2}$ are as in (9), then the results of the game do not change provided that the order of the four goals is kept intact. To see that this holds, it is sufficient to write out the payoff functions as piecewise-defined functions. Table 1 gives the matrix of utilities for the strategies of each parent providing between zero and three hours of care, broken into half-hour increments.

Note that the payoffs are all positive, with a higher number indicating a better outcome. The maximum possible payoff is 15, which happens for both parents when the parents are sharing equally (goal G4 contributes one to payoff) exactly three hours of care (goal G1 contributes ten to payoff). Because each parent is doing 1.5 hours of care, each is meeting goal G2 (contributing three to payoff), and each gets 50% of the possible payoff for goal G3 (personal time) because each is doing half of the maximum care he or she might have to do (this contributes one to payoff).

For each parent, the strategies of providing zero or a half-hour of care are dominated by the other strategies. After eliminating the dominated strategies, the utility of all the other strategies depends on the choice of the other player. There are three Nash equilibria – the equal-sharing outcome in which each parent does 1.5 hours of child care and the two splits in which one parent does two hours of care and the other does one hour. These outcomes are stable in the sense that neither player has an incentive to make a unilateral change in strategy. Only the equal-sharing outcome (1.5, 1.5) is Pareto optimal, meaning any other outcome would decrease the payoff for at least one player. This equal-sharing solution gets both players their best outcome of 15. Next I explore how parents reach one of these equilibria.

Repeated play

Because this situation is faced daily by parents while children are young, I assume that the parents are playing a repeated game, selecting a single pure strategy each time, and taking turns in the play. Here the analysis is inspired by Brams' *Theory of Moves* (Brams, 1994). The parents alternate their changes in strategy, with each parent in turn seeking to optimize his or her payoff within a column or row. Thus, for instance, once the column player

Table 1. Payoff matrix for the symmetric child care game with parameters as in (9).

		Mother's hours						
		0	0.5	1	1.5	2	2.5	3
Father's hours	0	(3, 3)	(4.5, 5.7)	(6, 8.3)	(7.5, 9.5)	(9, 10.7)	(10.5, 11.8)	(12, 13)
	0.5	(5.7, 4.5)	(7.5, 7.5)	(9, 10.2)	(10.5, 11.3)	(12, 12.5)	(13.5, 13.7)	(13.3, 13.2)
	1	(8.3, 6)	(10.2, 9)	(12, 12)	(13.5, 13.2)	(15, 14.3)	(14.8, 13.8)	(14.7, 13.3)
	1.5	(9.5, 7.5)	(11.3, 10.5)	(13.2, 13.5)	(15, 15)	(14.8, 14.5)	(14.7, 14)	(14.5, 13.5)
	2	(10.7, 9)	(12.5, 12)	(14.3, 15)	(14.5, 14.8)	(14.7, 14.7)	(14.5, 14.2)	(14.3, 13.7)
	2.5	(11.8, 10.5)	(13.7, 13.5)	(13.8, 14.8)	(14, 14.7)	(14.2, 14.5)	(14.3, 14.3)	(14.2, 13.8)
	3	(13, 12)	(13.2, 13.3)	(13.3, 14.7)	(13.5, 14.5)	(13.7, 14.3)	(13.8, 14.2)	(14, 14)

Note: Father's payoff is listed first. Nash equilibria are shaded.

has chosen a new strategy, she will have no incentive to change her strategy until the row player changes his strategy because she is at the maximum payoff within a fixed row.

Because each parent has seven different strategy choices, this game has 49 different ''starting positions'' or initial pairs of strategy choices by the two parents. As noted above, the strategies of zero hours and a half-hour are dominated for each player, but these are still considered as possible starting positions, both for completeness and because parents are not necessarily able to analyze the structure of the game. With a given starting position, either player can choose to initiate a series of moves that will terminate when both players are satisfied with the outcome (when neither player has an incentive to move unilaterally). This will happen at one of the Nash equilibria. A player is also not allowed to ''pass'' – if there is a unilateral move that improves the player's outcome, the player must make the move that gives the highest utility given his or her partner's current strategy choice.

To play this game, the parents start with any set of two strategy choices, for instance (0.5, 3), with the father doing a half-hour of care and the mother doing three hours of care (note there is a half-hour in which parents are providing care together). On the payoff matrix (Table 1) this is represented by row two and column seven, and these strategies provide a payoff of 13.3 to the father and 13.2 to the mother. Suppose that the father changes his strategy first. Looking along the seventh column (representing the fixed strategy choice of three hours by the mother) yields the highest possible payoff in this column for the father, which is row three, corresponding to increasing his contribution to an hour of care. The parents are then at the strategy pair (1, 3) with a payoff of (14.7, 13.3). Now the mother has an opportunity to change her strategy. Looking at the row corresponding to the father's strategy of one hour (row three) and choosing the strategy that maximizes her payoff yields column five which corresponds to reducing her contribution to two hours of care and provides a payoff of (15, 14.3). The father then has another opportunity to move, but he is getting a payoff of 15, his highest possible payoff. Thus (1, 2) will be a stable final outcome of the game, with neither parent having an incentive to change strategy unilaterally. To summarize, this game play was:

$$\begin{array}{ccccc} (0.5,3) & & (1,3) & & (1,2) \\ \text{Starting position} & \rightarrow & +0.5 \text{ hours father} & \rightarrow & -1 \text{ hours mother} \end{array}$$

If the mother switches strategies first with the same starting point, the following sequence occurs:

$$\underset{\text{Starting position}}{(0.5,3)} \rightarrow \underset{-0.5\ \text{hours mother}}{(0.5,2.5)} \rightarrow$$

$$\underset{+0.5\ \text{hours father}}{(1,2.5)} \rightarrow \underset{-0.5\ \text{hours mother}}{(1,2)}$$

With this starting point of (0.5, 3), no matter who goes first, the end result is that the mother is providing two hours of care to the father's one hour of care.

Starting position matters

There are some starting positions that allow different outcomes depending on which player goes first. Suppose the couple starts out providing equal care for the child with some overlap, say at position (2, 2). In this case, if the father goes first he immediately reduces to one hour, which is a stable position from which the mother does not wish to move. But if the mother goes first, she immediately reduces to one hour as well, which is a stable position from which the father does not wish to move. Analyzing the game in this way leads to a matrix which shows the final result of the game for all 49 different starting positions (see Table 2).

In 39% of the starting positions in Table 2 the final outcome is independent of which parent moves first, leading to the consideration of when a parent is actually able to impact the final outcome. Looking at the array of strategy choices, the starting positions for which this kind of agency is possible for both parents are the starting positions in which there is either vastly too much or starkly too little child care. The strategy matrix of Table 2 splits into four regions. In region I, both parents are doing at most one hour of child care. In region II, the mother is doing two to three hours of child care, while the father does zero to one. In region III, both the mother and the father are doing two to three hours of child care, and in region IV, the father is doing two to three hours of child care, while the mother does one to two. These regions are bounded by the cases in which one parent does exactly half of the care (1.5 hours). In region I there is a clear joint problem and whichever parent moves first to solve the problem by increasing the number of hours will do the majority of the child care. In region III, there is also a choice, because there is a surplus of hours being done, and the first person to reduce hours will reap the reward of that excess. Regions II and IV are mirror images of each other; in these regions approximately the right amount of care is occurring, but there is an imbalance in the care provided.

Table 2. Final stable outcomes of the symmetric child care game.

		Mother's hours						
		0	0.5	1	1.5	2	2.5	3
Father's hours	0	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(1.5, 1.5) or (1, 2)	(1, 2)	(1, 2)	(1, 2)
	0.5	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(1.5, 1.5) or (1, 2)	(1, 2)	(1, 2)	(1, 2)
	1	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(1.5, 1.5) or (1, 2)	(1, 2)	(1, 2)	(1, 2)
	1.5	(2, 1) or (1.5, 1.5)	(2, 1) or (1.5, 1.5)	(2, 1) or (1.5, 1.5)	(1.5, 1.5)	(1, 2) or (1.5, 1.5)	(1, 2) or (1.5, 1.5)	(1, 2) or (1.5, 1.5)
	2	(2, 1)	(2, 1)	(2, 1)	(1.5, 1.5) or (2, 1)	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(2, 1) or (1, 2)
	2.5	(2, 1)	(2, 1)	(2, 1)	(1.5, 1.5) or (2, 1)	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(2, 1) or (1, 2)
	3	(2, 1)	(2, 1)	(2, 1)	(1.5, 1.5) or (2, 1)	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(2, 1) or (1, 2)

Note: Final outcomes are listed as a pair of hours, with the father's listed first. When two possibilities are given, the first represents the outcome if the father moves first, the second if the mother moves first. Nash equilibria are in dark gray. The light gray cells are the cells from which it is possible to get to the Pareto optimal (equal-sharing) solution. The matrix is divided into four regions by the light gray cells, which are referred to starting with region I in the upper left and moving clockwise through regions II, III, and IV.

Collectively, parenting among dual-earner couples is currently taking place in region II – men are active participants, but women are still doing more. Much is made of the choices that working mothers have with respect to career and family. However, a mother starting in region II does not have any effective choices, since she cannot impact the final outcome. Without the existence of a surplus of care (or the pressures of a deficit in care) the choice is illusory. Indeed, neither the mother nor the father starting in this region has an escape from the foregone conclusion that the mother will provide the majority of the care. Debate may be ongoing about the sources and structure of power in the family – whether fathers have power due to their economic advantages or mothers have power due to maternal gatekeeping – but such power is irrelevant in region II of this game, because neither parent is able to impact the outcome.

In region II, both parents have a barrier that eliminates the possibility of moving to a different final outcome even if we allow alteration of the utility functions (thus changing the game). For the mother to create a change in the final outcome, she has to reduce her hours until total child care provided is at an unacceptable level (recall that in this model, outsourcing of child care is not an option). For the father to create change, he has to raise his total number of hours, reducing his own payoff until there is a surplus of care. These barriers seem qualitatively different. To cross her barrier, the mother is required to do something that directly risks her status as a good mother, although the possibility of outsourcing might change the impact of this decision and is worth exploring (see section ''Assumptions and extensions of this work''). The father, however, crosses his barrier by doing something that causes him personal inconvenience rather than public shame, and his status as a good father is at risk only if the additional care impacts his ability to be a good provider. In fact, if he provides more child care, he also may receive the benefit of being perceived as a father who is willing to go above and beyond the call of duty and sacrifice for his family. Looking at utilities confirms this analysis. Even if the parents are prohibited from dipping below their preferred minimal time in child care (one hour), the average payoff to mothers moving from region II to the far-right values of region I drops by 21% (12.9 to 10.2), whereas the average payoff to fathers going from the bottom values in region II to region III drops only by 4% (14.8 to 14.2).[1] Mothers starting in region II and seeking to increase their payoffs have no effective choices at all even if they are allowed to move in a manner that is outside the current rule structure, because the cost of creating change is too great. Fathers starting in this same region would pay a smaller cost to create change, but they have little incentive to do so, as they are already destined to get very close to their best outcome. Another way to frame the barriers to a move outside of region II is that it is a move from both parents ''doing gender'' in region II to ''undoing gender'' (Deutsch, 2007), and it seems that for this scenario there is a higher cost to the mother when she attempts to undo gender than there is for the father.

Remark on choice of hour parameters

Note that the choice of parameters $h_{P,1}$ and $h_{P,2}$ obviously makes a difference in the outcome of the game. The choice of $h_{P,1}$ controls the total hours of care that are needed, and the choice of $h_{P,2}$ sets a lower bound below which parent P will increase his or her hours. With the other parameters as in (9) but with $h_{P,2} = 1.5$ for each player P, for any starting point, the stable solution will be the Pareto-optimal equal sharing solution of 1.5 hours for each parent. For such parents there is no difficulty in sharing care equally, and this parameter selection will likely match reality for some parents. Both fathers and mothers have been increasing the amount of time they spend with their children in recent years (Sandberg and Hofferth, 2001), and one possible reason for this is the change in parenting norms (Coltrane, 1997). It may be that for some segments of society the average value of $h_{P,2}$ is growing, and if it grows large enough, then equal sharing of child care will no longer be as difficult to attain.

Considering a range of values for $h_{P,2}$ (minimal time desired by parent P) for any fixed value of $h_{P,1}$ (total care needed) gives a useful view of the parameter space. If $h_{P,2} = 0$ for both parents, then neither parent desires to spend time on child care, and there are final outcomes for all combinations of hours that add up to $h_{P,1}$. For $h_{P,2} \geq \frac{h_{P,1}}{2}$, there is only one final outcome; both parents will provide $h_{P,2}$ hours of child care. This is the case mentioned above, in which the parental goals around time spent with children (goal G2) make equal sharing of child care a foregone conclusion, eliminating any discrepancy between the desires of parents and the reality.

For the purposes of this analysis, the most interesting range for the parameter $h_{P,2}$ is $0 < h_{P,2} < \frac{h_{P,1}}{2}$, because this range produces a tension between the goal of parental free time (G3) and equal sharing of care (G4). In this parameter range, there are three possible outcomes that are stable – $(h_{P,2}, h_{P,1} - h_{P,2})$, $(h_{P,1} - h_{P,2}, h_{P,2})$, and $\left(\frac{h_{P,1}}{2}, \frac{h_{P,1}}{2}\right)$ – and the Pareto-optimal outcome of $\left(\frac{h_{P,1}}{2}, \frac{h_{P,1}}{2}\right)$ is the hardest to achieve. Note that as the value of $h_{P,2}$ approaches $\frac{h_{P,1}}{2}$ these three outcomes converge, and any difficulty the parents have achieving equal sharing disappears. In this paper the parameter value chosen was $h_{P,2} = 1$, but other values of $h_{P,2}$ with $0 < h_{P,2} < 1.5$ would lead to similar results to the ones presented here.

Implications

Even with identical goals that include a commitment to equally shared parenting, parents with goal parameters as in (9) are likely to end up with an imbalance in the amount of time spent on child care. It is fairly improbable that the equal-sharing solution, which provides the highest total payoff to the two parents combined, will be attained. To reach this solution, the starting position has to be one in which at least one parent does exactly half of the three hours of care needed and, if the starting position is not the equal-sharing solution itself, then equal sharing is achieved only when the parent doing the most care makes the first move (see Table 2).

It is fair to ask why the parents in this game would start at any position other than the equal-sharing one. There are many reasons why parents might start at positions that are suboptimal with respect to their goal of shared care, including the fact that equal sharing is ranked as the lowest-priority goal, which is an assumption of the model that can be questioned or altered. However, there is a clear tendency for mothers to spend more time with young infants (Bailey, 1994), landing new parents in region II of Table 2. Parenting experts from the postwar 1940's right up to today have insisted that a physically intimate, loving, and closely bonded relationship with the biological mother is as necessary for an infant as food, warmth, and safety (Plant, 2010; Sears, 2001), and even when involvement by the father is championed, it still appears to be less essential. Mothers who breastfeed must spend a great deal of time with their infants, especially very young infants; this leads to fathers having less time with their children initially (Gamble and Morse, 1993). Mothers are also more likely than fathers to take substantial leave from their jobs (Lammi-Taskula, 2008; Seward et al., 2002), which means that when those same mothers return to work, the parents are starting from a set of strategy choices in which the mother performs the bulk of the infant care. But even deeper than these observations, mothers may feel both a desire to have an equal child care partnership with fathers and a resistance to giving up the role of primary caregiver, so that the goal of shared care may not be as clear-cut as it appears. In order to uphold the ideals of egalitarian parenting while still maintaining primacy, such mothers may take up roles of gatekeepers and managers, setting standards for tasks and overseeing the work done by fathers, who may themselves collude in this gatekeeping (Allen and Hawkins, 1999).

However, the analysis of this game leads to a suggested course of action for parents who want to share care equally, despite the barriers to doing so. Such parents should consciously attempt to manipulate their starting position in the game and counteract the tendency for mothers to spend more time with young infants by having the father provide as much care as possible. This might happen through the father taking significant paternity leave, making sure the father gets a substantial amount of time taking care of the baby by himself even in early infancy, or striving to divide care consciously at the end of maternity leave. Couples who value equal sharing should start by equal sharing, even if biological, economic, or social factors make this choice difficult, because this solution may be challenging to achieve when parents start with an unequal division of labor. These recommendations are reflected in research that suggests that the availability of long periods of maternity leave may lead to a decrease in paid labor by women, but available paternity leave may mitigate that impact (Hook, 2010; Pettit and Hook, 2005). There is also evidence that even temporary decreases in women's paid work may lead to an increasingly traditional division of labor (Zvonkovic et al., 1996).

Graphical analysis and the continuous game

To picture what is happening as parents play this game, imagine the strategy choices of the parents laid out on a Cartesian grid with the father doing x hours child care and the mother doing y hours as in Figure 1. With any starting position (x, y), the parents are both strongly attracted to moving to the line $x+y=3$, which corresponds to meeting goal G1. Above or below that line, at least one parent has an incentive to move. When the starting position is above $x+y=3$, the mother will move toward the line $y=h_{c,2}=1$ while simultaneously remaining above the line $x+y=3$; when the father moves, he is trying to go toward the line $y=h_{r,2}=1$ while remaining above $x+y=3$. When the starting position is below $x+y=3$, either parent will immediately move to that line. Players can only move vertically (the mother) or horizontally (the father); they are not allowed to move diagonally (this would amount to players moving simultaneously in a coordinated way, which is not allowed in this game). For any starting point (x, y), the father will move to $\max(1, 3-y)$ because this will get him the fewest total hours spent in child care (goal G3) while meeting goals G1 and G2. Similarly, with starting point (x, y), the mother will play $\max(1, 3-x)$. In this case, goal G4 could

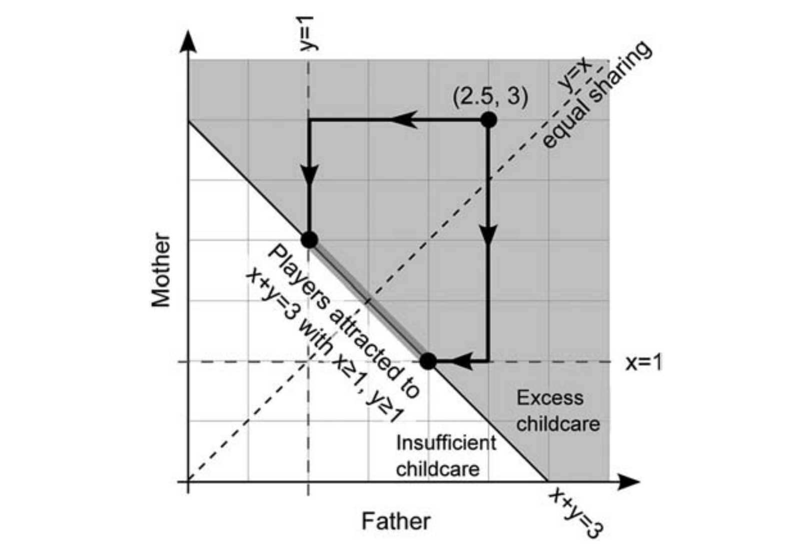

Figure 1. Graphical analysis of the symmetric child care game. The father's strategy choices lie along the horizontal axis, the mother's along the vertical axis. The shaded region represents strategy choices that result in an excess amount of child care (x + y>3). In the unshaded region (x + y<3), too little care is being done to meet goal G1. The starting point (2.5, 3) is shown, along with the series of moves that result in a final equilibrium of (1, 2) or (2, 1), depending on which parent goes first.

actually be eliminated and not change the outcome of the game (although it would change the payoffs).

In the continuous game, parents are allowed to make any choice with regard to their child care between zero and three hours (they are not restricted to just half-hour increments). In this situation the utility functions are continuous functions in two variables. The graph makes it clear what happens in the continuous case. In the continuous case the players are not restricted to intersections of grid, but a starting point can be placed anywhere at all. It remains true that starting at (x, y) the father will play strategy $\max(1, 3-y)$ and the mother will play $\max(1, 3-x)$, which means that every point on the line $x+y=3$ between (1, 2) and (2, 1) will be an equilibrium point, although (1.5, 1.5) remains the only Pareto-optimal solution. On the graph, this unique optimal solution is at the intersection between the line of equal sharing, $y=x$, and the line of required total child care, $x+y=3$.

Equal sharing of child care

Equal sharing was only possible in the previous game if at least one parent started out doing exactly half of the care and was only a certainty if both parents started out doing exactly half the care. Will increasing the value that parents place on the goal of equal sharing increase the probability that they end up at a stable outcome of equal sharing? There is no change in the final outcomes of the game until the goal of equal sharing (goal G4) moves above the goal of personal time (goal G3). When, for instance, $g_{P,4} = 2.5$ (for both parents), placing it in between goal G2 and G3, the outcomes change dramatically (see the payoff matrix of Table 3 and the matrix of final game results in Table 4).

In Table 4, there are four different stable final outcomes of this game, each corresponding to a strategy pair (i, i) for $i = 1.5, 2, 2.5, 3$. An interesting artifact of this is that most of the outcomes in this game involve the parents doing more work than is necessary. This is because the equal-sharing goal has been moved above the goal of personal time, so that both parents are willing to contribute excess time to child care. This results in equal sharing but also in a loss of efficiency. The parents can end up in a deadlock, each doing three hours of child care together with their partner because neither partner is willing to be the first to reduce hours. The graphical analysis of this game in Figure 2 shows that the parents are no longer as attracted to moving left (father) or down (mother) in order to minimize time; instead they both want to move toward the line $y = x$, and the equilibrium points all lie along that line. This may be a real phenomenon in equal-sharing couples and should be studied further. In the first pages of Deutsch's study of equal-sharing in parenting, she notes an example of this loss of efficiency in a couple who were both putting a child to bed at night until they realized that one parent would suffice (Deutsch, 1999: 2).

Gender differences

The assumption of identical goals for mothers and fathers is an egregious oversimplification, even if it does provide some useful information. Research on gender shows that men and women develop essential gender differences that impact their parenting (Coltrane, 1997; Risman, 2004; West and Zimmerman, 1987; Wilkie et al., 1998). Next I analyze two nonsymmetric games in which the mother and father do not have identical goals and payoff functions.

Table 3. Payoff matrix for the child care game with the equal-sharing goal elevated above the personal time goal.

		Mother's hours						
		0	0.5	1	1.5	2	2.5	3
Father's hours	0	(4.5, 4.5)	(5.8, 6.9)	(7, 9.3)	(8.3, 10.3)	(9.5, 11.2)	(10.8, 12.1)	(12, 13)
	0.5	(6.9, 5.8)	(9, 9)	(10.3, 11.4)	(11.5, 12.3)	(12.8, 13.3)	(14, 14.2)	(13.6, 13.4)
	1	(9.3, 7)	(11.4, 10.3)	(13.5, 13.5)	(14.8, 14.4)	(16, 15.3)	(15.6, 14.6)	(15.2, 13.8)
	1.5	(10.3, 8.3)	(12.3, 11.5)	(14.4, 14.8)	(16.5, 16.5)	(16.1, 15.8)	(15.7, 15)	(15.3, 14.3)
	2	(11.2, 9.5)	(13.3, 12.8)	(15.3, 16)	(15.8, 16.1)	(16.2, 16.2)	(15.8, 15.4)	(15.3, 14.7)
	2.5	(12.1, 10.8)	(14.2, 14)	(14.6, 15.6)	(15, 15.7)	(15.4, 15.8)	(15.8, 15.8)	(15.4, 15.1)
	3	(13, 12)	(13.4, 13.6)	(13.8, 15.2)	(14.3, 15.3)	(14.7, 15.3)	(15.1, 15.4)	(15.5, 15.5)

Note: Father's payoff is listed first. Nash equilibria are shaded.

Table 4. Final stable outcomes for the child care game with the equal-sharing goal elevated.

		Mother's hours						
		0	0.5	1	1.5	2	2.5	3
Father's hours	0	(3, 3)	(2.5, 2.5) or (3, 3)	(2, 2) or (3, 3)	(1.5, 1.5) or (3, 3)	(2, 2) or (3, 3)	(2.5, 2.5) or (3, 3)	(3, 3)
	0.5	(3, 3) or (2.5, 2.5)	(2.5, 2.5)	(2, 2) or (2.5, 2.5)	(1.5, 1.5) or (2.5, 2.5)	(2, 2) or (2.5, 2.5)	(2.5, 2.5)	(3, 3) or (2.5, 2.5)
	1	(3, 3) or (2,2)	(2.5, 2.5) or (2,2)	(2, 2)	(1.5, 1.5) or (2,2)	(2, 2)	(2.5, 2.5) or (2,2)	(3, 3) or (2, 2)
	1.5	(3, 3) or (1.5, 1.5)	(2.5, 2.5) or (1.5, 1.5)	(2, 2) or (1.5, 1.5)	(1.5, 1.5)	(2, 2) or (1.5, 1.5)	(2.5, 2.5) or (1.5, 1.5)	(3, 3) or (1.5, 1.5)
	2	(3, 3) or (2, 2)	(2.5, 2.5) or (2, 2)	(2,2)	(1.5, 1.5) or (2, 2)	(2, 2)	(2.5, 2.5) or (2, 2)	(3, 3) or (2, 2)
	2.5	(3, 3) or (2.5, 2.5)	(2.5, 2.5)	(2, 2) or (2.5, 2.5)	(1.5, 1.5) or (2.5, 2.5)	(2, 2) or (2.5, 2.5)	(2.5, 2.5)	(3, 3) or (2.5, 2.5)
	3	(3, 3)	(2.5, 2.5) or (3, 3)	(2, 2) or (3, 3)	(1.5, 1.5) or (3, 3)	(2, 2) or (3, 3)	(2.5, 2.5) or (3, 3)	(3, 3)

Note: Final outcomes are listed as a pair of hours, with the father's listed first. The first result listed occurs when the father goes first, the second when the mother goes first. One result is listed when it doesn't matter who goes first. Nash equilibria are shaded.

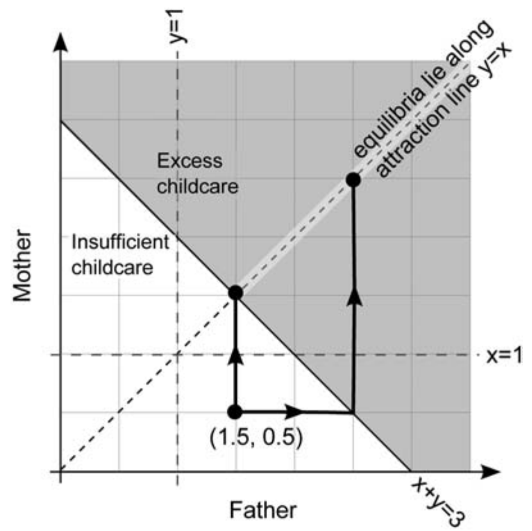

Figure 2. Graphical analysis of the child care game with the equal-sharing goal elevated. In this game, parents want to be in the shaded region (meeting goal G1), but this time in the shaded region they do not move left or down to minimize their time; instead they move toward the line *y* = *x* in order to share care with a partner. The starting position of (1.5, 0.5) is marked along with the moves that give the final equilibrium solutions of (2.5, 2.5) or (1.5, 1.5).

Fathers, mothers and quality time

A generation or two ago, fathers were not as involved with their children as they are today (Coltrane, 1997). It is a fairly recent phenomenon for men to hold spending time with their children as an essential parenting goal (Smith, 2009). For the next game, the father's goal of quality time is removed by setting $g_{r,2} = 0$ with all other parameter values as in the original list (9). It would also be reasonable to argue that the goal of equal sharing should be removed for both parents, but doing so for one or both parents does not change the outcomes of this game. Note that changing $g_{r,2}$ does not change anything about the mother's payoffs or behavior; those remain as in the original game with parameters as in (9). The payoff matrix for this new game is found in Table 5 and the results matrix in Table 6.

The outcome of this game leans, not surprisingly, toward results in which the mother does the bulk of the care. This game has five stable final outcomes of (2, 1), (1.5, 1.5), (1, 2), (0.5, 2.5), and (0, 3).

Table 5. Payoff matrix for the child care game with the father's goal of quality time (G2) removed.

		Mother						
		0	0.5	1	1.5	2	2.5	3
Father	0	(3, 3)	(4.5, 5.7)	(6, 8.3)	(7.5, 9.5)	(9, 10.7)	(10.5, 11.8)	(12, 13)
	0.5	(4.2, 4.5)	(6, 7.5)	(7.5, 10.2)	(9, 11.3)	(10.5, 12.5)	(12, 13.7)	(11.8, 13.2)
	1	(5.3, 6)	(7.2, 9)	(9, 12)	(10.5, 13.2)	(12, 14.3)	(11.8, 13.8)	(11.7, 13.3)
	1.5	(6.5, 7.5)	(8.3, 10.5)	(10.2, 13.5)	(12, 15)	(11.8, 14.5)	(11.7, 14)	(11.5, 13.5)
	2	(7.7, 9)	(9.5, 12)	(11.3, 15)	(11.5, 14.8)	(11.7, 14.7)	(11.5, 14.2)	(11.3, 13.7)
	2.5	(8.8, 10.5)	(10.7, 13.5)	(10.8, 14.8)	(11, 14.7)	(11.2, 14.5)	(11.3, 14.3)	(11.2, 13.8)
	3	(10, 12)	(10.2, 13.3)	(10.3, 14.7)	(10.5, 14.5)	(10.7, 14.3)	(10.8, 14.2)	(11, 14)

Note: Father's payoff is listed first. Nash equilibria are shaded.

Table 6. Final stable outcomes for the child care game with the father's goal of quality time (Goal G2) removed.

		Mother						
		0	0.5	1	1.5	2	2.5	3
Father	0	(2, 1) or (0, 3)	(2, 1) or (0, 3)	(2, 1) or (0, 3)	(1.5, 1.5) or (0, 3)	(1, 2) or (0, 3)	(0.5, 2.5) or (0, 3)	(0, 3)
	0.5	(2, 1) or (0.5, 2.5)	(2, 1) or (0.5, 2.5)	(2, 1) or (0.5, 2.5)	(1.5, 1.5) or (0.5, 2.5)	(1, 2) or (0.5, 2.5)	(0.5, 2.5)	(0, 3) or (0.5, 2.5)
	1	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(2, 1) or (1, 2)	(1.5, 1.5) or (1, 2)	(1, 2)	(0.5, 2.5) or (1, 2)	(0, 3) or (1, 2)
	1.5	(2, 1) or (1.5, 1.5)	(2, 1) or (1.5, 1.5)	(2,1) or (1.5,1.5)	(1.5, 1.5)	(1, 2) or (1.5, 1.5)	(0.5, 2.5) or (1.5, 1.5)	(0, 3) or (1.5, 1.5)
	2	(2, 1)	(2, 1)	(2, 1)	(1.5, 1.5) or (2, 1)	(1, 2) or (2, 1)	(0.5, 2.5) or (2, 1)	(0, 3) or (2, 1)
	2.5	(2, 1)	(2, 1)	(2, 1)	(1.5, 1.5) or (2, 1)	(1, 2) or (2, 1)	(0.5, 2.5) or (2, 1)	(0, 3) or (2, 1)
	3	(2, 1)	(2, 1)	(2, 1)	(1.5, 1.5) or (2, 1)	(1, 2) or (2, 1)	(0.5, 2.5) or (2, 1)	(0, 3) or (2, 1)

Note: Final outcomes are listed as a pair of hours, with the father's listed first. The first result listed occurs when the father goes first, the second when the mother goes first. There is only one result listed when the result is the same no matter who goes first. Nash equilibria are shaded.

Despite the tendency toward the mother providing more care, it is possible to end up with the father doing more of the care if the game starts with the father doing more. This is because both parents place paramount importance on getting the required total number of child care hours. Because of this, the father cannot improve upon his payoff from the strategy pair (2, 1) unilaterally. This might seem like an unlikely place for a negotiation between such a pair of parents to start, but it could mimic the situation of a mother going ''on strike'' in an effort to change the dynamic around child care. This strategy is even recommended in a popular book on parenting (Cockrell et al., 2008).

How much work is there?

Mothers and fathers may not estimate their involvement with child care equally; in many cases fathers are more likely to believe that care is being shared equally, while mothers are more likely to believe they have primary responsibility for childrearing (Milkie et al., 2002). Factors contributing to this imbalance in perception may include the desire for egalitarian partnerships being ahead of the reality of such partnerships and leading fathers to overestimate their contributions, the tendency of people to overestimate their own contributions to burdensome tasks, and the perceived fairness (or lack thereof) in the allocation of family labor, particularly by mothers (Kamo, 2000). It is possible to model a discrepancy using the current game structure by building in differing perceptions of need. Perhaps one parent believes a child needs constant parental attention and another thinks an hour in a playpen or in front of a TV with minimal interaction is fine. Or perhaps each parent makes a different mental estimate of how much time is needed to care for a child. For this variation of the game, the assumption is that the father thinks two hours of child care are needed and the mother thinks four hours are needed (as would be the case if the father were making an underestimate and the mother making an overestimate). In this case, with $h_{r,1} = 2$ and $h_{c,1} = 4$ and all other parameters as in (9), the only stable final outcome of the game is $(1, 3)$ and all starting points lead toward that conclusion. The mother believes she is doing most of the work required (75%), while the father believes he is doing half of the amount of care that is truly necessary, either not noticing the mother's additional contribution or believing that it is in excess of the needed care (and is thus voluntary and desired).

One obvious way for the mother to improve her outcome is for the parents to come to a negotiated agreement about the hours of work that need to be done. A second, less obvious way out of the difficulty is for both parents to increase the importance of the equal-sharing goal (goal G4). With this adjustment the players end up sharing parenting, as in Table 4, at the cost of doing excessive amounts of total care.

A third possibility is for the father to change his goals, either eliminating his personal time goal (G3) or lowering it below the sharing goal (G4). This can be thought of as the father either increasing the importance of sharing or decreasing the importance of personal time. In this game, most starting points lead to stable cycles rather than Nash equilibria. See Tables 7 and 8 for game results with $g_{r,3}=0$. Similar results hold whenever $g_{r,4}>g_{r,3}$ with $h_{r,1}=2$ and $h_{c,1}=4$ and other parameters as in (9). One example of such a cycle would be a game that starts with fewer than four hours of child care being provided. For example, if the game starts at (1, 1), play would proceed as follows:

(1, 1) Starting position, both parents providing one hour of care;
(1, 3) Mother increases to three hours, meeting her goal of four total hours;
(3, 3) Father increases to three hours, meeting his goal of sharing care;
(3, 1) Mother decreases to one hour, still meeting her goal of four total hours;
(1, 1) Father decreases to one hour, still meeting his goal of two total hours.

The couple has now returned to the starting position, and the cycle continues like this indefinitely. This solution becomes an equal-sharing cycle, in which first one parent is doing more work and then the other. There are, in fact, two different stable cycles for this game (see Table 8), as well as one Nash equilibrium point at (2, 2). In a graphical analysis of the game (Figure 3), it becomes clear that the cycling is produced by the mother's attraction to the line $x+y=4$ and the father's attraction to the line $y=x$.

Conclusion

This paper provides a possible mechanism by which couples with an intention to share the work of caring for children equally end up with an unequal division of labor. Equal sharing is a difficult equilibrium to reach and the period directly after the birth of a child often features the mother providing substantially more hands-on care than the father. In the repeated game utilized by this model, it is not possible for a parent providing more care to withdraw this care if doing so moves the parents under the required amount

Table 7. Payoff matrix for the child care game with the father estimating two hours of care needed, the mother estimating four hours of care, and the father eliminating the personal time goal.

		Mother								
		0	0.5	1	1.5	2	2.5	3	3.5	4
Father	0	(1, 3)	(3.4, 5.4)	(5.8, 7.8)	(8.1, 8.6)	(10.5, 9.5)	(10.4, 10.4)	(10.3, 11.3)	(10.1, 12.1)	(10,13)
	0.5	(4.9, 4.1)	(7.5, 6.8)	(9.9, 9.1)	(12.3, 10)	(12.1, 10.9)	(12, 11.8)	(11.9, 12.6)	(11.8, 13.5)	(11.6,13.1)
	1	(8.8,5.3)	(11.4,7.9)	(14,10.5)	(13.9,11.4)	(13.8,12.3)	(13.6,13.1)	(13.5, 14)	(13.4, 13.6)	(13.3,13.3)
	1.5	(11.1, 6.4)	(13.8, 9)	(13.9, 11.6)	(14, 12.8)	(13.9, 13.6)	(13.8, 14.5)	(13.6, 14.1)	(13.5, 13.8)	(13.4, 13.4)
	2	(13.5, 7.5)	(13.6, 10.1)	(13.8, 12.8)	(13.9, 13.9)	(14, 15)	(13.9, 14.6)	(13.8, 14.3)	(13.6, 13.9)	(13.5, 13.5)
	2.5	(13.4, 8.6)	(13.5, 11.3)	(13.6, 13.9)	(13.8, 15)	(13.9, 14.9)	(14, 14.8)	(13.9, 14.4)	(13.8, 14)	(13.6, 13.6)
	3	(13.3, 9.8)	(13.4, 12.4)	(13.5, 15)	(13.6, 14.9)	(13.8, 14.8)	(13.9, 14.6)	(14, 14.5)	(13.9, 14.1)	(13.8, 13.8)
	3.5	(13.1, 10.9)	(13.3, 13.5)	(13.4, 14.9)	(13.5, 14.8)	(13.6, 14.6)	(13.8, 14.5)	(13.9, 14.4)	(14, 14.3)	(13.9, 13.9)
	4	(13, 12)	(13.1, 13.4)	(13.3, 14.8)	(13.4, 14.6)	(13.5, 14.5)	(13.6, 14.4)	(13.8, 14.3)	(13.9, 14.1)	(14, 14)

Notes: Father's payoff is listed first. Unique Nash equilibrium is shaded. In contrast to previous games, this matrix includes up to four hours of care by each parent.

Table 8. Outcomes for the child care game with the father estimating two hours of care needed, the mother estimating four hours of care, and the father eliminating the personal time goal.

		Mother								
		0	0.5	1	1.5	2	2.5	3	3.5	4
Father	0	(2, 2) or C1	C2 or C1	C1	C2 or C1	(2, 2) or C1	C2 or C1	C1	C1	C1
	0.5	(2, 2) or C1	C2 or C1	C1	C2 or C1	(2, 2) or C1	C2 or C1	C1	C1	C1
	1	(2, 2) or C1	C2 or C1	C1 to (1, 3)	C2 or C1	(2, 2) or C1	C2 or C1	C1 to (3, 3)	C1	C1
	1.5	(2, 2) or C2	C2	C1 or C2	C2 to (1.5, 2.5)	(2, 2) or C2	C2 to (2.5, 2.5)	C1 or C2	C1 or C2	C1 or C2
	2	(2, 2)	C2 or (2, 2)	C1 or (2, 2)	C2 or (2, 2)	**(2, 2)**	C2 or (2, 2)	C1 or (2, 2)	C1 or (2, 2)	C1 or (2, 2)
	2.5	(2, 2) or C2	C2	C1 or C2	C2 to (1.5, 1.5)	(2, 2) or C2	C2 to (2.5, 1.5)	C1 or C2	C1 or C2	C1 or C2
	3	(2, 2) or C1	C2 or C1	C1 to (1, 1)	C2 or C1	(2, 2) or C1	C2 or C1	C1 to (3, 1)	C1	C1
	3.5	(2, 2) or C1	C2 or C1	C1	C2 or C1	(2, 2) or C1	C2 or C1	C1	C1	C1
	4	(2, 2) or C1	C2 or C1	C1	C2 or C1	(2, 2) or C1	C2 or C1	C1	C1	C1

Notes: Outcome is given by noting whether parents end up at the unique Nash equilibrium of (2, 2) in dark gray, or in one of the two stable cycles, C1 in light gray or C2 in medium gray. In contrast to previous games, this matrix includes up to four hours of care by each parent.

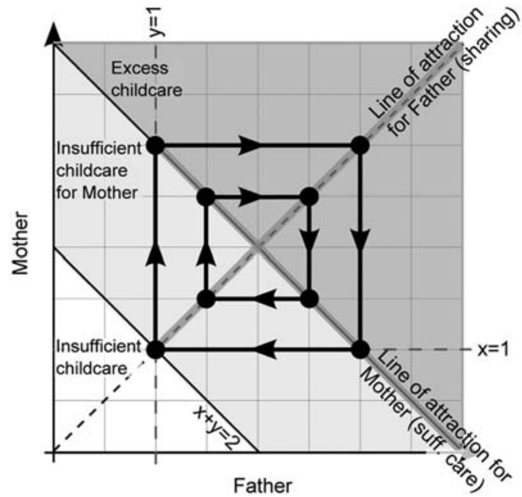

Figure 3. Graphical analysis for the child care game with the father estimating two hours of care are needed, the mother estimating four, and the father eliminating the personal time goal. The mother wants to be in the dark gray area in order to meet goal G1, but the father meets goal G1 by being in either of the two shaded areas. Provided that the father is meeting goal G1, he is attracted to the line $y = x$, but the mother is most attracted to the line $x + y = 4$. Two cycles of behavior are seen in this situation, as shown above. The point at which the line $x + y = 4$ and the line $y = x$ intersect is the only Nash equilibrium.

of care. Thus, if a couple wants to share the care of a young child equally, it is important to start by sharing care, which means planning carefully before the birth of a child.

It is also worthy of note that it is generally agreed that mothers are doing more child care work than fathers, which puts couples in region II of Table 2, from which position neither parent has a choice about the ultimate stable resolution of the game. In order for both parents to be able to impact the outcome, the parents must start with either a clear excess or a clear deficit in total hours of care provided.

Parents can increase the chances of an equal-sharing solution by increasing the importance of their goal of sharing child care. This produces the complication of making the goal of sharing care so important that parents devote more time than is necessary to child care, and the resulting loss of efficiency could have a negative impact on other areas of life. A future study could investigate whether families with a high commitment to equal sharing experience the efficiency loss predicted by the game as in Tables 3 and 4.

This game-theoretic model also provides a method for exploring variations between parents, including differences based on gender. Lowering or eliminating a father's goal of quality time increased the likelihood of ending up at an equilibrium in which the mother did most of the child care. If the mother and father have identical goals but estimate the amount of care needed differently, with the father underestimating and the mother overestimating, this again leads to a pattern in which the mother does the bulk of the care. Interestingly, a father who is willing to sacrifice his free time can improve the situation for the mother by eliminating that goal, but when he does, the parents end up in an equal sharing cycle in which first one parent and then the other are doing more care.

Assumptions and extensions of this work

In the construction of the utility functions for this game, I have made assumptions about the goals and preferences of parents which may well be incorrect. However, this model is only meant to shed light on possible mechanisms and interactions, not to give an accurate representation of parental behavior or to have predictive power. Surveys of parental goals paired with time diaries for both parents could be used to construct utility functions that match observed choices and would improve the usefulness of the model. Data from families headed by lesbians or gay men could be used to develop a model testing the conclusion that families that start with a differential in child care end up with that as a stable final outcome without the complication of differences in gender. The model presented in this paper could also be altered to allow for multitasking by parents (Craig, 2006) or outsourcing to non-parental child care.

The model presented here is one of a non-cooperative game, which means that there is no negotiation between the parents. Many economists use non-cooperative game theory in modeling interactions in marriage because true negotiation requires the creation of an enforceable contract, which is not realistic in the private realm of marriage, although some cooperative models have essentially used the marriage itself as the contract with a threat-point of divorce. However, there is some evidence that people are inclined to uphold even non-enforceable contracts (Irlenbusch, 2006). Assuming this, cooperative game theory could provide a framework for considering explicit negotiation between parents about child care, and that may provide a better model of the structure of the negotiation in some situations. Note that in the model presented in this paper, allowing parents to negotiate coordinated moves would create the possibility of diagonal moves in the matrix of the game, and thus it might be possible to negotiate a Pareto-optimal solution from any starting point.

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Note

1. If moves are allowed from anywhere in region II to anywhere in region III or region I, the situation becomes even more uneven. The father's average payoff moving from region II to region III actually increases by 11%, whereas the mother's payoff moving from region II to region III decreases by 43%.

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